T_N = normalized temperature, $T/\frac{\epsilon}{\kappa}$
T_R = reduced temperature, T/T_c
u_{11} = numerical factor, Equation (13)
v_c = critical volume, cc./g.-mole
x_i = mole fraction of species i
z_c = critical compressibility factor, $P_c v_c / RT_c$

Greek Letters

α = constant, Equation (3)
β = constant, Equation (9)
γ = ratio of heat capacities, C_p/C_v
δ = diffusivity modulus, $M^{1/2}/P_c^{2/3}T_c^{5/6}$
$\mathcal{D}_{ij}$ = coefficient of diffusion for species i and j
ϵ = maximum energy of interaction between two particles, erg.
κ = Boltzmann constant, 1.3805×10^{-16} erg./°K.
λ = thermal conductivity modulus, $T_c^{1/6}M^{1/2}/P_c^{2/3}$
λ' = pseudothermal conductivity modulus
μ = viscosity, g./sec. cm.
π = pressure, atm.
σ = collision diameter, Å.
$\Omega^{(1,1)}[T_N]$ = collision integral for diffusivity
$\Omega^{(2,2)}[T_N]$ = collision integral for thermal conductivity

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Manuscript received May 15, 1964; revision received August 24, 1964; paper accepted August 24, 1964.

Particle Flow Patterns in a Fluidized Bed

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A modified thermistor anemometer probe was used to measure solid particle flow patterns in a 9.5-in. diameter fluidized bed. The probe was calibrated and found to measure the direction and mass velocity of the silica-alumina cracking catalyst used. Flow patterns were determined at heights of 6.5, 12.5, 18.5, and 24.5 in. above the support plate with superficial air velocity from 0.25 to 1.00 ft./sec. and bed depths of 14 to 30 in.

Radial flow patterns were found not to be symmetrical about the axis of the bed but were similar at angles of 60 deg. This indicated that there were six similar particle flow pattern sections around the column axis.

The percentage of the cross-sectional area of the fluidized bed through which the particle flow is upward was found to be approximately 60%. There was no definite dependence of this area on any of the variables studied.

The catalyst bulk circulation rate, defined as the average mass velocity of the particles at a given cross section in a fluidized bed without regard to the direction of flow, was determined for each fluidization condition and correlated as a function of the superficial air velocity and the ratio of the distance above the support plate to the height of the fluidized bed.

It has generally been proposed that solids travel down a fluidized bed close to the wall and up through the center of the bed with cross flow between the center and the wall taking place at several elevations in the bed. These speculations about the circulation pattern are based almost entirely on visual observation at the surface of the bed, and the travel of particles within the bed remains in doubt.

Particle velocities along the column wall were studied by Toomey and Johnstone (*16*) and Massimilla and Westwater (9) using high-speed photographic techniques. Particle velocities up to 6 ft./sec. were noted, with very abrupt changes in direction taking place. Side motion of the particles was relatively mild.

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Leva and Grummer (5) distributed dyed particles on the top of a fluidized bed and measured the time required for the uniform distribution of the dyed particles to take place in the bed, as determined by visual observation.

Others (*2, 4, 6, 10, 12, 13, 14*) have used tagged particles to study the rate of mixing of solids.

No direct measurements of particle flow patterns in the interior of a fluidized bed have been found in the literature, and this investigation probably represents the first reported investigation of these flow patterns.

GENERAL METHOD

Hot-wire anemometers have been widely used to measure point velocities in a moving stream of fluid (*11*). In order to measure flow patterns in a fluidized bed, the principal of an anemometer was employed, but the probe was designed to have directional characteristics. A thermistor probe was constructed in which two thermistors were placed adjacent to one another at the end of the probe. One thermistor was electrically heated just as it would be for an anemometer. However, instead of measuring the change in resistance of the heated thermistor, the resistance change of the adjacent thermistor was measured. This second thermistor will be called the *indicating thermistor*.

The operation of the thermistor probe is based on the transfer of energy as heat from the heater thermistor to the indicating thermistor. If the mass flow of fluid (fluid in this sense is meant to include particles in suspension) is in such a direction that it passes over the heater thermistor first and then over the indicating thermistor, as shown in (*a*) of Figure 1, there will be a net transfer of heat to the indicating thermistor, causing a lowering of its resistance. If the mass flow of fluid, however, is in the opposite direction, that is from the indicating thermistor to the heater thermistor, as shown in (*b*) of Figure 1, the only heat transferred to the indicating thermistor will be that due to radiation and conduction, and the lowering of resistance of the indicating thermistor will be much less.

The difference between the heat transferred in the first case and that transferred in the second case is proportional to the mass flow of fluid, since the radiant heat transfer should be the same in each case and cancel out. The alignment of the probe resulting in a maximum change in the thermistor resistance will be an indication of the direction of flow.

APPARATUS AND MATERIALS

Thermistor Probe and Related Measuring Apparatus

The thermistor probe consisted of two wafer types of thermistors attached by their leads to the end of a ⅛-in. diameter ceramic holder. A drawing of the end of the probe is shown in Figure 2. The ceramic holder was contained in a length of ¼-in. diameter aluminum tubing used to position the probe in the fluidized bed.

The temperature of the heater thermistor was maintained at a value of approximately 100°C. by adjusting the voltage and current so that the resistance of the thermistor was always 90 ohms. A 500-ohm 10-w. resistor was used as a current limiting resistor to prevent the thermistor from burning itself out. This limiting resistor was necessary because of the negative temperature coefficient of resistance which is characteristic of thermistors.

The indicating thermistor was connected in a Wheatstone's bridge. An electronic circular chart recorder, having a full-scale range of 7 mv., was used to measure the signal from the bridge, and an analogue computer was used to integrate the signal, in order to obtain a time average value of the thermistor resistance.

Equipment for Calibration of the Probe

A diagram of the equipment used to calibrate the probe is shown in Figure 3. An 8-ft.-long vertical 1.5-in. I.D. smooth brass tube served as a vertical test section through which air

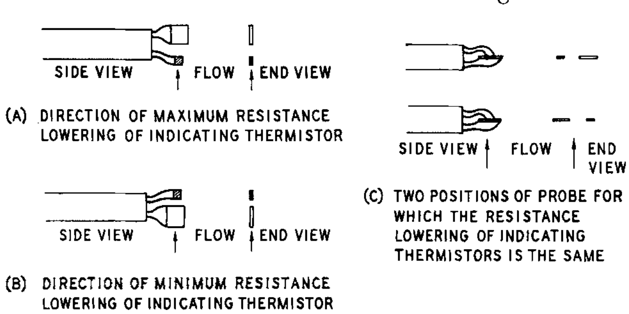

Fig. 1. Different orientations of the thermistor probe.

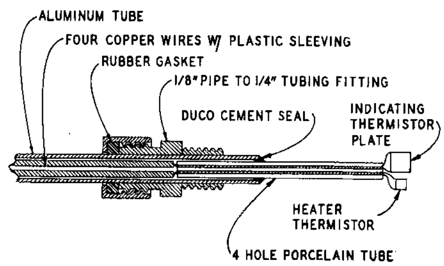

Fig. 2. Detailed drawing of thermistor probe.

alone or a mixture of air and catalyst was continually conveyed. The thermistor probe was placed 2 ft. from the top of the tube.

The air or catalyst-air mixture entered the test section through a 2-ft. radius bend of 1.5-in. I.D. flexible steel tubing and left the top of the test section through another length of steel tubing connected to a cyclone.

The catalyst escaped from the bottom of the cyclone into a vertical plexiglass tube 3.5-in. I.D., which served as a catalyst storage hopper.

The catalyst was delivered from the storage hopper to the air stream by a horizontal 1.5-in. diameter 5-in.-long steel auger with a speed range of 0 to 20 rev./min.

The flow rate was controlled manually by a 1-in. gate valve and measured by a bank of three rotameters.

Equipment for the Fluidization Study

A diagram of the fluidization equipment is shown in Figure 4. The column used for the fluidized bed was a plexiglass tube 4 ft. long and 9.5-inches I.D. with ¼-in.-thick walls. The support for the bed was a ¼-in.-thick type 304 stainless steel porous plate having a mean pore opening of 35 μ. The entrance section to the bed immediately below the support plate was a polyethylene funnel having an angle of 60 deg.

Taps for the probe were placed at heights 6.5, 12.5, 18.5, and 24.5 in. above the support plate. At each level four holes were placed 30 deg apart.

The air source for the fluidized bed was the same turbo compressor used for the calibration of the thermistor probe.

The solid material used for the fluidized bed was silica-alumina spent fluid cracking catalyst. The particle size distribution and physical properties of this material are given in Table 1.

EXPERIMENTAL PROCEDURE

The thermistor probe was positioned and the voltage applied to the heater thermistor was adjusted so that its temperature was 100°C. The system was allowed 45 min. to reach a steady state before any data were taken. The time required for establishment of steady state was determined by measuring the flow pattern at one point in the bed as a function of time. It was found that 30 min. after initial start up the flow pattern would remain the same.

The thermistor probe was first positioned with the heater thermistor vertically below the indicating thermistor. The millivoltage produced by the bridge was integrated for an interval of at least 5 min. and an integral time average value of the millivoltage determined. The integral value was expressed in terms of the scale reading of the potentiometer recording the signal from the Wheatstone's bridge and is proportional to the average resistance of the thermistor during that time interval.

The thermistor probe was then turned over so the heater thermistor was vertically above the indicating thermistor. The integration was repeated for the probe in this position, and another integral time average value was determined.

The second average scale reading obtained was subtracted from the first, and the difference, expressed as scale difference (S.D.), was recorded as a measure of the magnitude and direction of the particle flow. A positive value of the scale difference indicated upward flow and a negative value downward flow.

A minimum of twenty points was determined for each profile.

TABLE 1. PHYSICAL PROPERTIES OF SILICA-ALUMINA CATALYST

Particle size distribution

On 140 mesh	1%
On 200 mesh	16
On 325 mesh	63
Through 325 mesh	16
Average bulk density	0.7 g./cc.

Calibration of the Probe with Mass Velocity

Measurements were taken with the thermistor probe positioned in the 8-ft.-long calibrating section for a series of runs in which air alone was used and a series in which different catalyst air mixtures were used.

Five radial points were determined across the 1.5-in. diameter tube for each air velocity with duplication of each point. The scale differences were weighed on the basis of the fraction of the cross-sectional area which they represented and an average value calculated.

The main variables which should affect velocity measurements with the thermistor probe in a fluidized bed aside from the physical properties of the medium are the air velocity, the particle velocity, and the fluidized bed density or fraction of the bed volume occupied by solid particles.

The scale differences obtained from the probe measurements were plotted as a function of the total mass velocities. The air mass velocity ranged from 40 to 3,470 lb./hr.-sq. ft. and the catalyst mass velocity from 865 to 4,060 lb./hr.-sq. ft. The total range of mass velocities studied was 40 to 7,300 lb./hr.-sq. ft.

The part of the calibration curve covering the lower range of mass velocities was determined with air only. This was necessary since there is a minimum fluid velocity, termed the "choking velocity" (*16*), below which it is impossible to entrain solids in a fluid stream. The part of the calibration curve which was determined with both air alone, and air with entrained solids is presented on an expanded scale in Figure 5. Within the experimental accuracy of the data, there is no evidence of any effect on the thermistor probe measurement, by the addition of solid catalyst, other than that caused by an increase in mass velocity.

Since the thermistor probe measurements were correlated as a function of the total mass velocity and the density of the silica-alumina catalyst used in this investigation has a settled bulk density approximately 700 times the density of air, the contribution of air to the mass velocities measured by the thermistor probe in a fluidized bed is negligible. Hence, in a fluidized bed, the thermistor probe will essentially measure the solid particle mass velocity. Additional evidence that the probe was not effected by the air in the fluidized bed was obtained from the profiles themselves. An average value of the recorded mass velocity for each set of profiles was determined. If the air flow contributed to the probe measurements, then each profile should show a net flow upward in the bed. Since this was not the case, but instead the net flow was zero, it substantiated the conclusion that in a fluidized bed only particle flow would be measured.

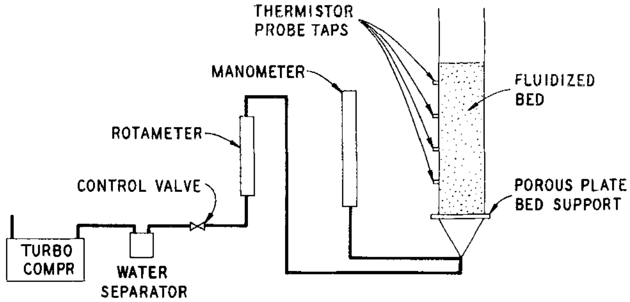

Fig. 4. Diagram of fluidization equipment.

RESULTS

Interpretation of Radial Profiles

Radial profiles of the particle flow pattern in a fluidized bed were obtained with two thermistor probes on projections forming a 30-deg. angle with the center of the column. A total of forty-eight radial profiles were determined with each of the two thermistor probes. Four settled bed depths of 12, 16, 20, and 24 in., four superficial air velocities of 0.25, 0.55, 0.72, and 1.00 ft./sec., and four probe heights of 6.5, 12.5, 18.5, and 24.5 in. were studied in all possible combinations.

Very rapid changes in particle velocity occur, and although these changes were detected by the probe, only the average value for a 5 to 10 min. time interval was recorded. Therefore, flow patterns obtained were for bulk movement of particles rather than instantaneous particle velocities.

Asymmetry of the Fluidized Bed

The particle flow pattern for a given radial profile was found to be symmetrical with the axis of the bed. Early in the investigation it was found that radial profiles could not always be reproduced. Different profiles would appear after the fluidizing air had been shut off and then turned on again, although the flow pattern at any one position in the bed was reproduced for 6 hr. if the bed was not shut down during this time. This was attributed to the bed not being symmetrical about its axis, and when the fluidized bed was started up again, this asymmetry would shift in its relation to the column axis.

This was investigated by positioning three other thermistor probes at the same column height at angles of 30, 60, 90 deg. with the first probe, the angle being that formed by the two probes and the axis of the column. Since each profile was found to be symmetrical with the center of the fluidized bed, the minimum frequency of appearance of the same average velocity in an arc about the column axis at a given distance from the axis was two times per revolution. Thus, the fluidized bed would have to be symmetrical to any vertical plane through the column

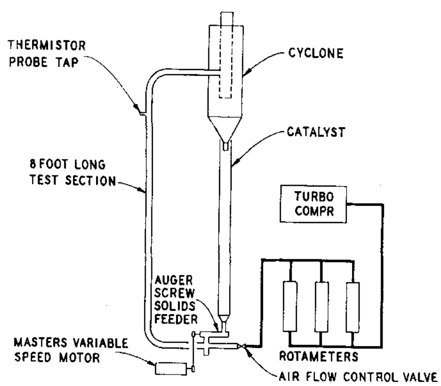

Fig. 3. Diagram of thermistor probe calibration equipment.

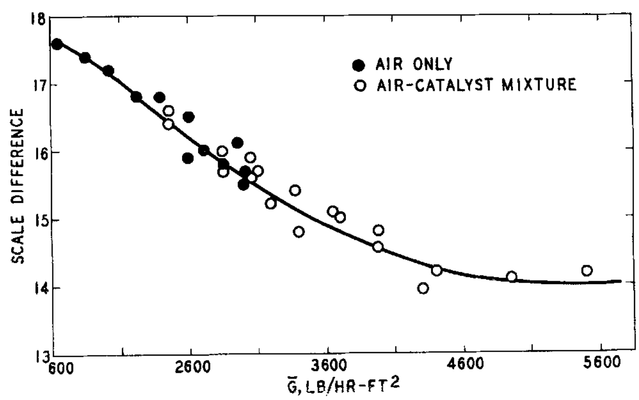

Fig. 5. Section of thermistor probe calibration for which both air alone and air-catalyst mixtures were measured.

axis. Likewise, the frequency of appearance of the same average velocity would have to be some multiple of two, such as two, four, six, etc. Owing to the size of the cross-sectional area of the fluidized bed, any frequency above six was considered unlikely.

If the frequency were two, then none of the probes would provide the same profiles, and if four, only the two probes at 90-deg. angles would show the same profile. If the frequency were six, then the probes at 60-deg. angles with one another would show the same profiles.

Measurements were made at a distance of 4.25 in. from the axis of the column with the four probes, and the two sets of probes at 60-deg. angles were found to have similar readings. This would agree with a frequency of appearance of six. A repetition of the flow pattern in an arc about the column axis was therefore shown to exist. A shift in the relationship of the repetitive flow pattern with the column wall on successive start ups of the fluidized bed would then account for the nonreproducibility of the radial profiles.

A study was made to determine as near as possible the nature of the repetition of the flow pattern. It was found that a sine function would predict the thermistor probe measurements obtained at any angular position 4.25 in. from the fluidized bed axis.

For this study a series of measurements were made in which the four probes were positioned in different arrangements in the probe taps of 0, 30, 60, and 90 deg., and a wave function of the form

$$\text{S.D.} + k_1 = k_2 \sin f\,(\theta + \theta_c)$$

was applied to the data. The scale difference (S.D.) was corrected by the constant k_1 in order to make the amplitude k_2 of the sine function symmetrical about the mean between the maximum and minimum scale differences obtained. The frequency of appearance f for 1 cycle of this wave function in the full 360 deg. about the column axis was determined to be six. The probe position θ was corrected by θ_c, to account for the rotation of the fluidized bed about the column axis on successive start ups of the fluidized bed.

Figure 6 is a representation of how this sine function would account for the probe measurements obtained. It was assumed that the amplitude of the sine function was equal to the maximum observed measurements made in this study and that the frequency of appearance f was six. In arriving at this plot of the scale difference S.D. as a function of $\theta + \theta_c$, the measurement of the scale difference obtained by probe A was located on the curve and θ_c calculated. The other probe positions were corrected for θ_c and found to fit the relationship very well.

The time required for the determination of a single profile (approximately 4 hr.) made it necessary to limit

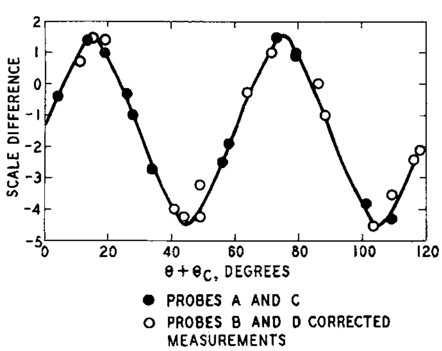

Fig. 6. Representation of measurements made with four thermistor probes at 4.25 in. from column by a sine function with frequency factor of 6.

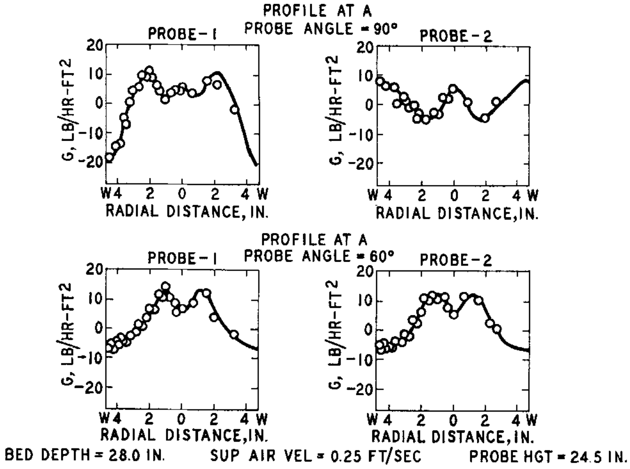

Fig. 7. Comparison of profiles obtained under the same fluidization conditions.

the number of probes used in the remainder of this investigation to two.

The asymmetry of the flow pattern for a given cross section of the bed is evident from four sets of radial profiles obtained with the two thermistor probes. The two thermistor probes were inserted in the column probe taps at angles of 30, 60, and 90 deg. with one another and the profiles obtained. Two sets of these profiles obtained at 60- and 90-deg. angles are shown in Figure 7.

The type of fluidized bed support plate used is probably a very critical factor in determining the asymmetry of a fluidized bed. In this study, with a porous plate, the fluidizing air was uniformly introduced into the bed of particles. This probably accounted for the shift in the asymmetry with the column axis on successive start ups of the fluidized bed.

If the support plate was designed so that the air was not uniformly distributed to the bed of solids, but was localized at very definite positions at the base of the bed, then the asymmetry with the column axis would probably be fixed. The solid particles would then be moving up in the fluidized bed at the position where the air is introduced and down at all other positions. Such a support plate would be a nonporous plate containing a number of holes through which the air passes. A perfectly symmetrical bed would then be expected to result if a single hole were present at the center of the plate.

With a porous plate, however, the positions at which the air concentrates into bubble streams moving up through the fluidized bed is a matter of chance, and thus the asymmetry with the column axis is not fixed.

Selected Radial Profiles

A total of forty-eight radial profiles were determined with each of the two thermistor probes. Only a number of selected profiles, illustrating the nature of the information obtained, will be presented. A complete set of data can be found in (7) and preliminary data in (8).

Even though the fluidized bed was definitely asymmetrical, a sufficient number of profiles were determined to enable the selection of three sets of profiles of which each profile in a given set seemed to be at the same projection from the axis in the fluidized bed. Four profiles showing the effect of superficial air velocity on the particle flow pattern in a fluidized bed are shown in Figure 8. The profiles are very similar and show only one peak between the center of the bed and the wall of the column. The radial distance at which the peak exists varied from 1.0 in. at a superficial air velocity of 0.25 ft./sec. to 2.75 in. at a superficial air velocity of 1.0 ft./sec.

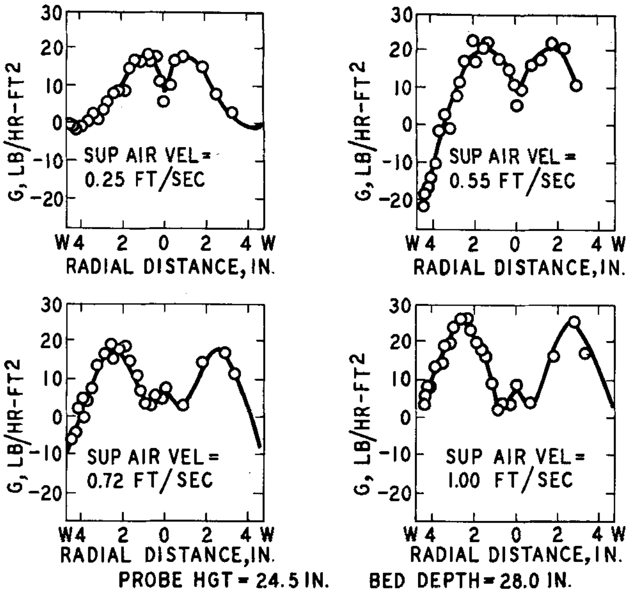

Fig. 8. Radial profiles for different superficial air velocities.

There is a general opinion that bubbles of air passing through a fluidized bed grow in size by coalescence of smaller bubbles into larger bubbles. Since the cross section, shown in Figure 8, is 24.5 in. above the support plate, the bubble size should be relatively large. It is conceivable, then, that the regions of particles moving up the column at velocities greater than 10 lb./hr.-sq. ft., shown in Figure 8, represent regions where the bubbles have concentrated, and that the passage of bubbles in a fluidized bed establishes the particle flow pattern.

Following this same reasoning that the upward particle flow is located at the same point in a fluidized bed as the bubble flow, the profiles indicate that at a low air velocity the bubbles coalesce into one air bubble at a lower height above the support plate than with a higher air velocity where one bubble of a larger size will be formed at a higher position above the support plate. Thus, at the same height in a fluidized bed, two air bubbles will have a larger separation for a higher air velocity than for a lower air velocity.

The profiles in Figure 9 indicate the effect of the position above the support plate on the radial particle velocity profile. Five peaks were found in the profile at 6.5 in. above the support plate, three peaks at 12.5 in., and two peaks at 18.5 in. This again was in agreement with the concept that the particle flow pattern is determined by the size and number of the bubble streams, which grow in size and diminish in number as they proceed upward through a fluidized bed.

The bed depth seemed to have little effect on the particle velocity profile obtained at a given height above the support plate, as shown in Figure 10. Baumgarten and Pigford (*1*) and Yasui and Johanson (*17*) found that the bed depth above a bubble in a fluidized bed had no effect on its size. Since the number of bubbles and their size are related at a given air velocity, it can be said that the bed depth has no effect on the number of peaks in the profiles for each bed depth, adding additional support to the concept that the particle flow pattern is determined by the air bubbles.

The low particle mass velocities which were obtained should probably be discussed at this point. These may appear to be inconsistent when compared with estimates of particle mass velocities which have been estimated previously. Lewis, Gilliland, and Bauer (*3*) have estimated solid mass velocities, based on a heat transfer model, of the order of 100,000 lb./hr.-sq. ft. Talmor and Benenati (*15*) using tagged particles have estimated solid circulation rates, based on a mixing model of 200 to 20,000 lb./hr.-sq. ft. Their results were in agreement with those reported earlier by Leva and Grummer (*5*) using a similar technique.

Consider a fluidized bed just past the point of incipient fluidization; if it is assumed that there is complete random mixing of the particles, at any one point over a period of time the direction and particle velocity would be under a constant state of change. The average mass velocity for the time interval in any given direction could very well be zero. The technique of estimating particle velocities of the above mentioned authors would not indicate a value of zero but would indicate the absolute magnitude of the particle velocity regardless of the direction of travel. The technique used in this study would indicate a particle mass velocity of zero, since it would take into account the direction of travel.

Consider now a fluidized bed somewhat past incipient fluidization. From the theory proposed by Toomey and Johnstone (*16*), the additional gas not required for the initial fluidization of the particles will pass through the

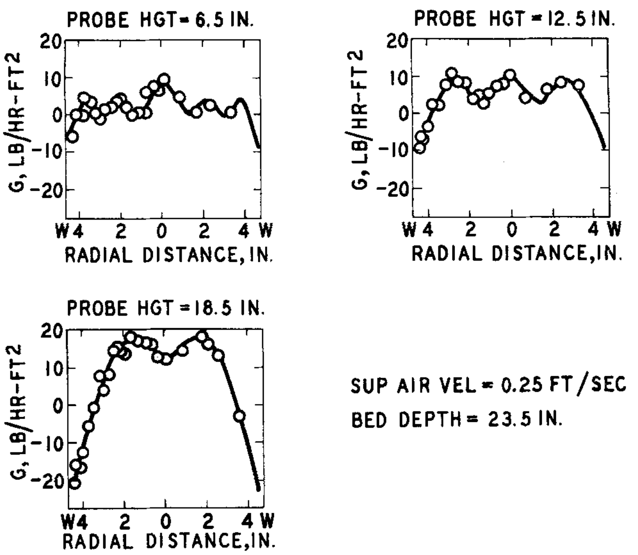

Fig. 9. Radial profiles for different positions above the support plate.

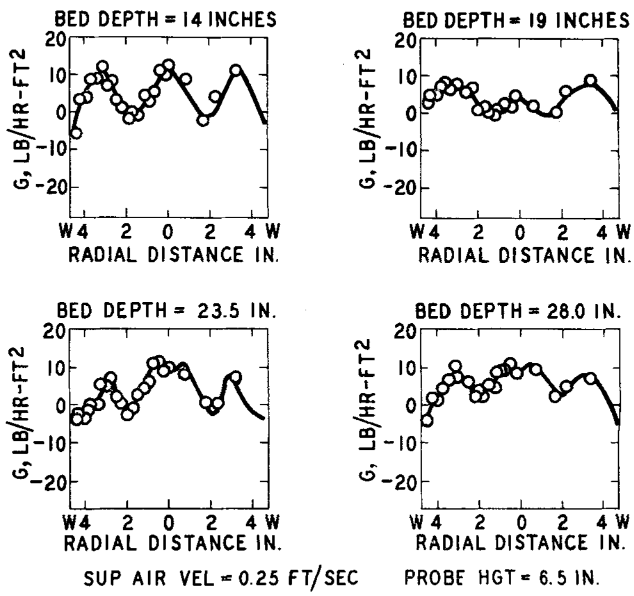

Fig. 10. Effect of bed depth on radial profiles.

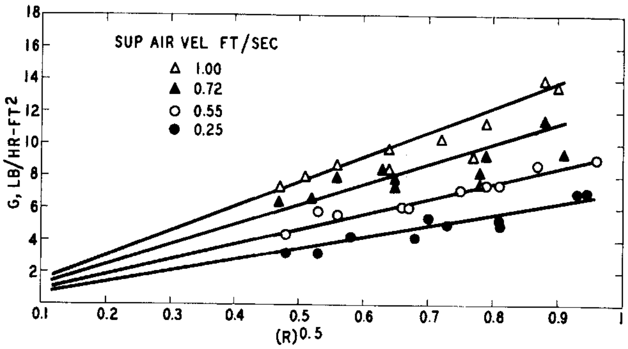

Fig. 11. Effect of position above the support plate on cataylst bulk circulation rate.

bed in the form of air pockets or bubbles. It is proposed here that the bubble streams which are formed will impart a defined overall movement to the already randomly mixed bed, and that this movement will increase the turbulence or rate of mixing of the particles in the bed. In other words there will be a slow movement of the randomly mixed particles through the bed. The particle flow patterns reported here are those of the movement of a well-mixed bed caused by the introduction of bubble streams to the bed.

Once again upon comparison of the previously reported particle circulation rates with those reported here, the previously employed techniques would give no indication of this slow bulk movement of particles since direction of movement was not considered. The technique employed in this study does not measure the particle velocities in the randomly mixed phase of the bed but only the movement of this phase in the vessel itself. The technique employed here did give an indication of the high instantaneous particle velocity at any one point in the bed and also of the rapid changes in velocity and direction that took place. The technique itself or the instrumentation employed was not adequate to measure instantaneous velocities.

The apparent inconsistencies between estimates of particle mass velocities reported in other studies and those reported here are not true inconsistencies. They are instead mass velocities of two different things. There have been no investigations which report results directly comparable to those disclosed here.

Percentage of the Bed Cross-Sectional Area Attributable to Upward and Downward Flow of Particles

The percentage of the cross-sectional area of the fluidized bed through which the particle flow was in an upward direction was calculated for each condition studied. The average of all the calculated areas was 60%. The standard deviation, however, was $\pm$ 10%, indicating the variables studied may have some effect on the percentages of the cross-sectional area through which the flow is up or down. No attempt was made to correlate this data, since it is doubtful that the two profiles made at each bed condition would show the true percentages of upward and downward flow of the particles.

There was some indication from the data that the percentage of the cross-sectional area having catalyst flowing upward increases with velocity and decreases with the height above the support plate. The percentages varied so widely, though, that a more extensive study could show the opposite to be true.

It can be stated, however, with fair certainty, that more than 50% of the cross-sectional area of the fluidized bed had particles flowing upward. In order, then, for the mass velocity of the particles to be the same in both directions in the fluidized bed, the particles must move up in the bed at a higher linear velocity in a less dense particle and air mass.

Correlation of the Catalyst Bulk Circulation Rate

The catalyst bulk circulation rate is defined as the average mass velocity of the particles, at a given position in a fluidized bed, without regard to the direction of flow. This average mass velocity G' was determined from the following expression in which the particle mass velocity is an absolute value:

$$G' = \frac{\sum_{i=1}^{i=10} (|G_i|A_i)_I + \sum_{i=1}^{i=10} (|G_i|A_i)_{II}}{2}$$

The average bulk circulation rate G' was determined for each set of radial profiles obtained for the various fluidization conditions. The three sets of radial profiles, obtained under the same fluidization conditions, an expanded bed depth of 28.5 in. and a superficial air velocity of 0.25 ft./sec., at a probe position 24.5 in. above the support plate, showed the same bulk circulation rate. The values of G' were 7.1, 7.3, and 7.4 lb./hr.-sq. ft. which indicates that the asymetry of the fluidized bed had no effect on the bulk circulation rate obtained from the thermistor probe measurements. The bulk circulation rate was also checked under a different set of fluidization conditions, an expanded bed depth of 23.5 in. and a superficial air velocity of 0.55 ft./sec., at a probe position 12.5 in. above the support plate. The values of G' determined from the two sets of radial profiles under these conditions were 7.1 and 7.0 lb./hr.-sq. ft.

The bulk particle circulation rate G' was correlated as a function of the superficial air velocity u and the ratio R of the distance above the support plate to the height of the fluidized bed. Both of these factors were found to affect G' by an exponential function.

The effect of the position above the support plate on the catalyst bulk circulation rate is illustrated in Figure 11. The final expression, for the effect of superficial air velocity and the ratio of the distance above the support plate to the height of the fluidized bed was found to be $G' = 14(uR)^{0.5}$.

The data scatter was not excessive, since the average deviation of $\pm$ 0.9 lb./hr.-sq. ft. represents an error of only 0.17 in the thermistor probe scale differences.

This correlation of the bulk circulation rate of particles in a fluidized bed shows that the average bulk velocity of the particles increases rapidly with increase in air velocity, at velocities just above the minimum fluidization velocity. but the rate of increase in particle mass velocity decreases as the reciprocal of the square root of the superficial air velocity.

If heat transfer in a fluidized bed is assumed proportional to G', where G' is the bulk mass velocity of the particles in a fluidized bed, then the heat transfer should also be proportional to $G^{0.5\alpha}$, where G is the mass velocity of the fluid, since G is proportional to the superficial air velocity u. Leva (*4*) reported values for the individual film coefficient h between the fluidized bed and the column wall or the wall of a heat exchanger placed within the fluidized bed. These heat transfer coefficients are proportional to G^{β}, where G is the superficial air mass velocity. The values of β, reported by several investigators, ranged from 0.3 to 0.6. This indicates the heat transfer coefficient may be directly proportional to the particle mass velocity, that is $\alpha = 1$, and that the difference in the reported values of β may be due to a difference in the relationship between the particle mass velocity and the superficial air velocity for different particle systems.

The correlation also shows the bulk circulation rate of the particles to increase with the distance above the support plate, and that the rate of increase in the mass velocity decreases as the reciprocal of the square root of the distance above the support plate.

This indicates that if the desired rate of mixing of particles being introduced into a fluidized bed is high, the particles should be introduced at the top of the fluidized bed. If the desired rate of mixing is low, the particles should be introduced into the bottom of the fluidized bed.

The minimum fluidization velocity for the silica-alumina catalyst used in this study is less than 0.02 ft./sec. and was therefore neglected in the correlation. A correlation of the bulk circulation rate of particles in a fluidized bed with superficial air velocity should be expressed in terms of $(u - u_{mf})$.

Another factor which should probably appear in the correlation is the bed depth. The data showed no apparent dependence on the bed depth, but this may be due to the fact that the lowest bed depth studied of 12 in. was so high that the effect of bed depth was within the experimental error of the measurements. If the effect of bed depth follows a relationship similar to that of the other variables studied, the greater effect on the particle mass velocity would occur at very low bed depths, which were not included in this investigation.

A fluidized bed of particles different than that used in this study, both in terms of particle size and type of material, may possess markedly different bulk circulation rates. Also, a different diameter, fluidized bed, or particle support plate would probably possess somewhat different characteristics.

SUMMARY

The ability of a modified thermistor anemometer probe to indicate direction as well as magnitude of material flow has been demonstrated by measuring particle flow patterns in a fluidized bed. The accuracy of the profiles obtained is based mainly on the consistency of the data, since there are no other published results of similar studies with which to compare these results. The probe was calibrated for only one system, air and silica-alumina cracking catalyst. There is no apparent reason why it shouldn't function for other solid particle systems, but this has yet to be proved.

A simple solid particle flow pattern, as generally thought to exist in the past, was shown to be a poor description of what happens. Particles did not flow up the center of the bed and down the wall of the column, but instead they appeared to flow up in those areas where the bubbles rise and flow down in the surrounding regions. The bubble flow pattern appears to establish the particle flow pattern. The bed height had little effect on the flow pattern at any given distance above the bed support, whereas the air velocity did.

Interpretations of flow patterns in fluidized beds other than the one studied should not be done. The effect of different support plates and bed diameters was not studied. These should be important factors in establishing the particle flow.

No single study of this nature could ever answer all of the mysteries of particle flow in a fluidized bed. It is hoped that this study has added a little more to the knowledge of fluidization, and that it serves not as a deterrent but as an inducement to study further the movement of particles in a fluidized bed.

ACKNOWLEDGEMENT

The authors wish to express their deep appreciation to the University of Maryland for making this study possible. Appreciation is extended to the Atlantic Refining Company for supplying the silica-alumina catalyst. The authors are also indebted to the E. I. duPont de Nemours and Company for the financial assistance which they provided through two summer Research Grants and to those with the Phillips Petroleum Company who assisted in preparing the manuscript.

NOTATION

- Ai = area of the ith annulus, in.^{2}
- f = frequency of appearance per revolution of the same flow pattern in an arc about the column axis, deg.$^{-1}$
- G = mass velocity of the solid particles, lb./hr.-sq. ft.
- G_i = average mass velocity for the ith annulus, lb./hr.-sq. ft.
- G' = average mass velocity for the total cross-sectional area, lb./hr.-sq. ft.
- $\overline{G}$ = total mass velocity of the air and solid particles
- i = number of the annulus section
- k_1, k_2 = constants
- S.D. = scale difference on recorder measuring bridge signal
- R = ratio of the distance above the support plate to the fluidized bed height
- u = superficial air velocity, ft./sec.
- U_{mf} = minimum fluidization velocity, ft./sec.
- α, β = constants
- θ = angle formed between any probe and probe I, deg.
- θ_c = correction factor for the shift of the flow pattern between beds, deg.
- I = profile made with probe I
- II = profile made with probe II

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Manuscript received March, 11, 1963; revision received June 15, 1964; paper accepted June 16, 1964. Paper presented at A.I.Ch.E. Buffalo meeting.